Unstructured finite element models of cardiac electrophysiology using a deal.II-based library

DEPARTMENT of MATHEMATICS

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Introduction

Organ-scale finite element models of cardiac electrophysiology require high spatial resolution to capture sharp propagating wavefronts, resulting in computationally expensive tasks. This poster presents a library specifically designed to address these demanding simulations. The library's routines support the use of linear, and quadratic tetrahedral elements. Moreover, activation of the whole heart involves models that span the thick muscular heart walls and cable-like fibers of the cardiac conduction system, which includes the bundle of his and the Purkinje network. We addressed that by enabling coupling between meshes with different codimension. By enabling such coupling, the library aims to contribute to a more comprehensive understanding of the heart's intricate electrical behavior.

Governing equations for electrophysiology problem

$$\beta\left(C_m \frac{\partial V}{\partial t} + I_{ion}(V, \vec{w}\ ) + I_s\right) = \nabla \cdot (\,\boldsymbol{\sigma}\,\nabla V) \text{ on } \Omega$$

$$\frac{\partial \vec{w}}{\partial t} = \vec{g}(\vec{w}\ , V)$$

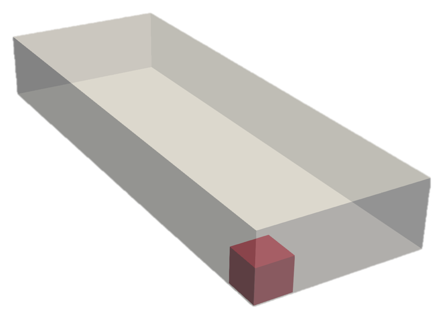
V - cell transmembrane potential difference
I_{ion} - outward cell transmembrane ionic current
I_s - initial stimulus current
β - ratio between the membrane area and the tissue volume;
C_m - cell membrane capacitance.
t - time
$\vec{w}$ - gating and ion flow variables
$\boldsymbol{\sigma}$ - conductivity tensor

Numerical approach

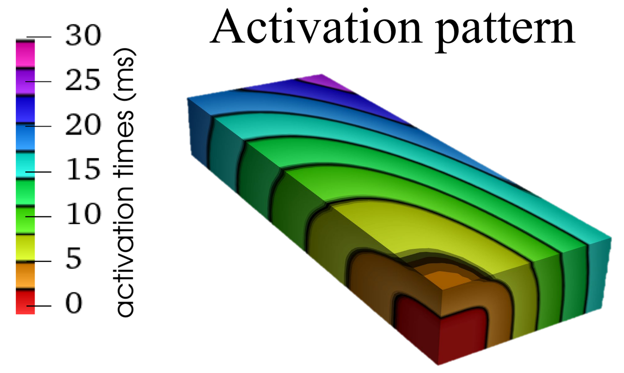
- Finite element formulation with linear and quadratic elements.
- First order Implicit-Explicit (IMEX) temporal scheme SBDF1.
- Forward Euler method to solve the system of ODEs associated with the ionic model.
- Conjugate gradient solver along multigrid preconditioner to solve linear problem associated with the PDE.

Benchmarking: p-refinement and h-refinement

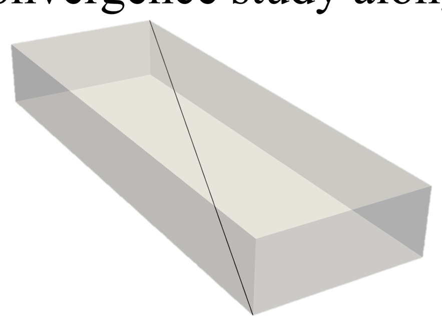
Initial stimulus region in red

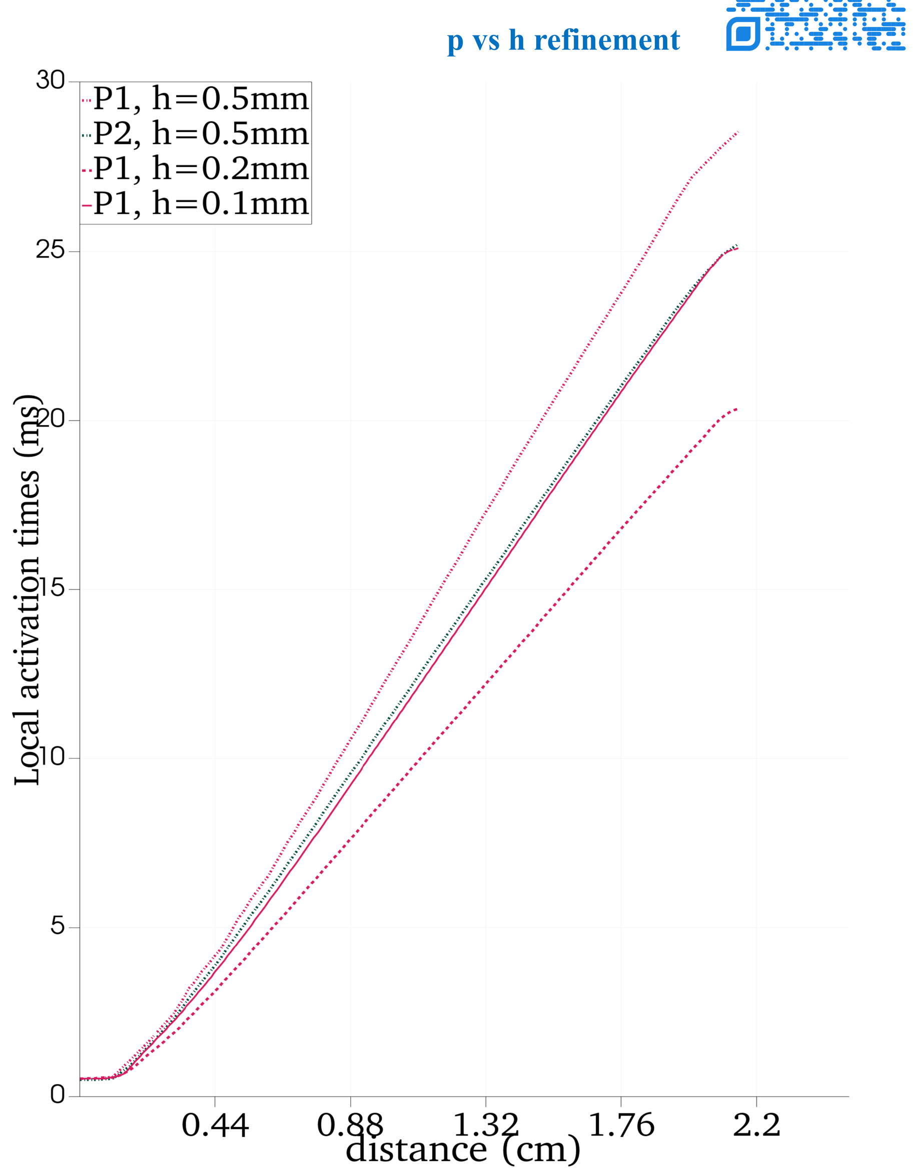


Activation pattern



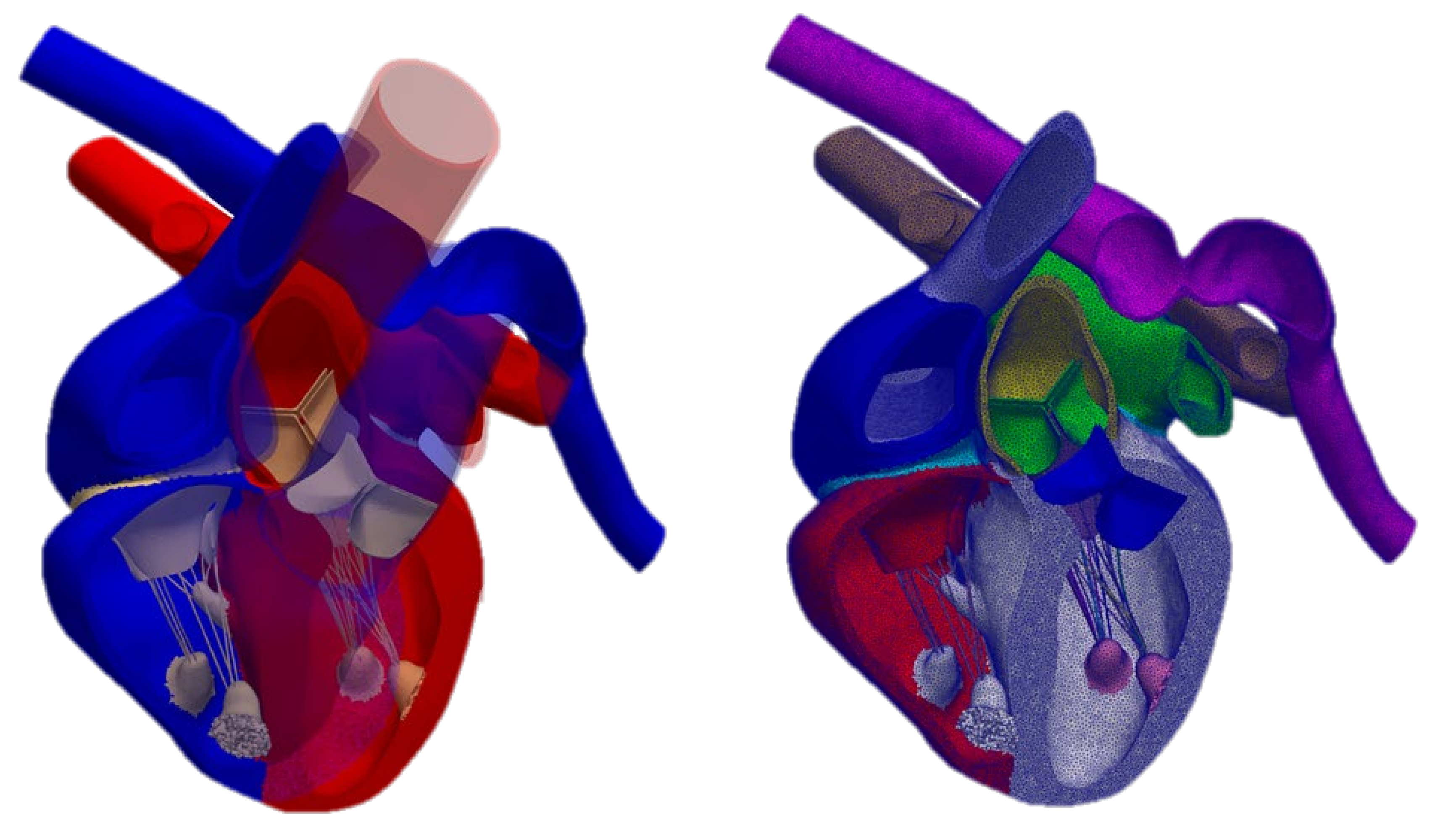
Convergence study along line



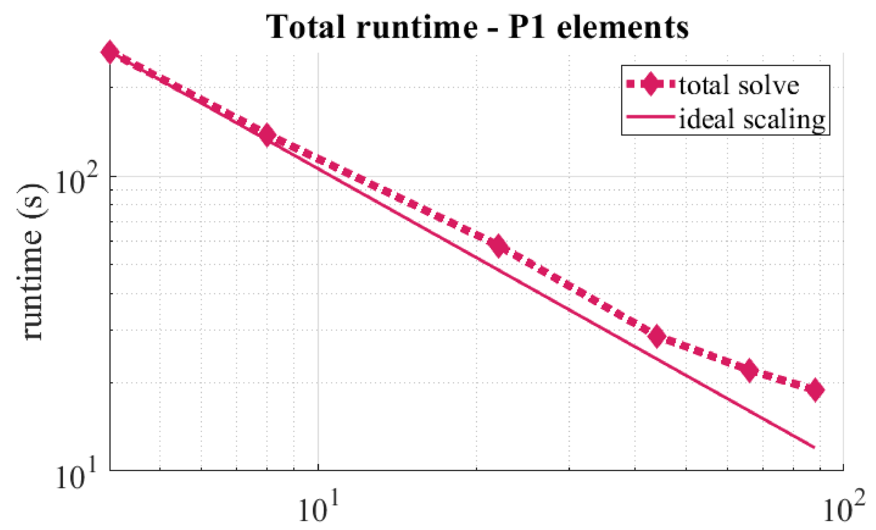


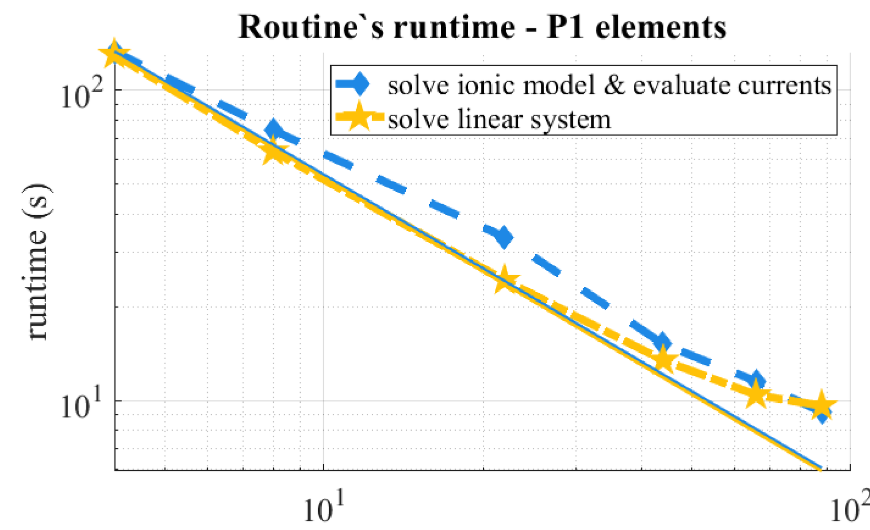
Runtime for solving whole heart mesh using linear and quadratic elements scales with number of processors

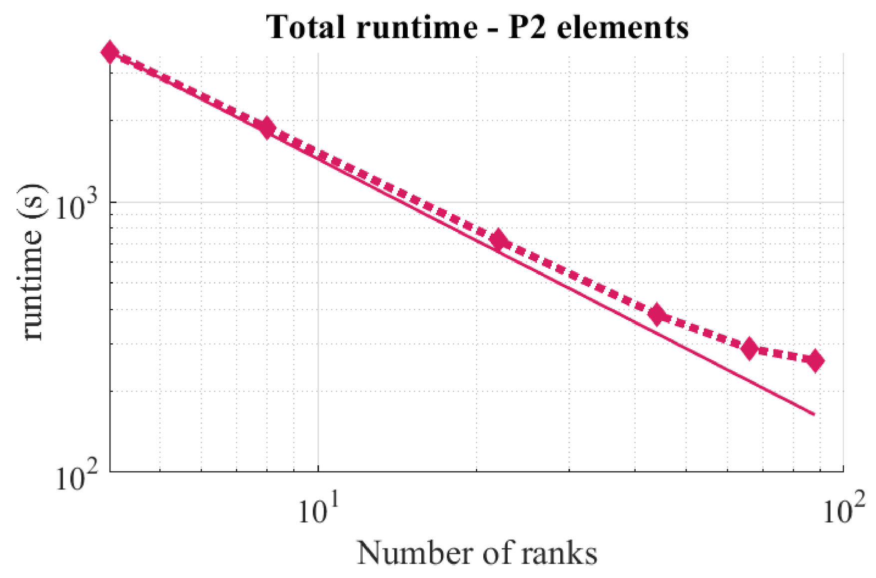
- Simulations performed in the heart mesh shown below using linear and quadratic elements.
- The total number of degrees of freedom were kept fixed.
- Increased the number of cores per node, keeping a balanced number of cores per node whenever possible.
- We activated a region in the right atrium and ran 100 timesteps of the simulation.

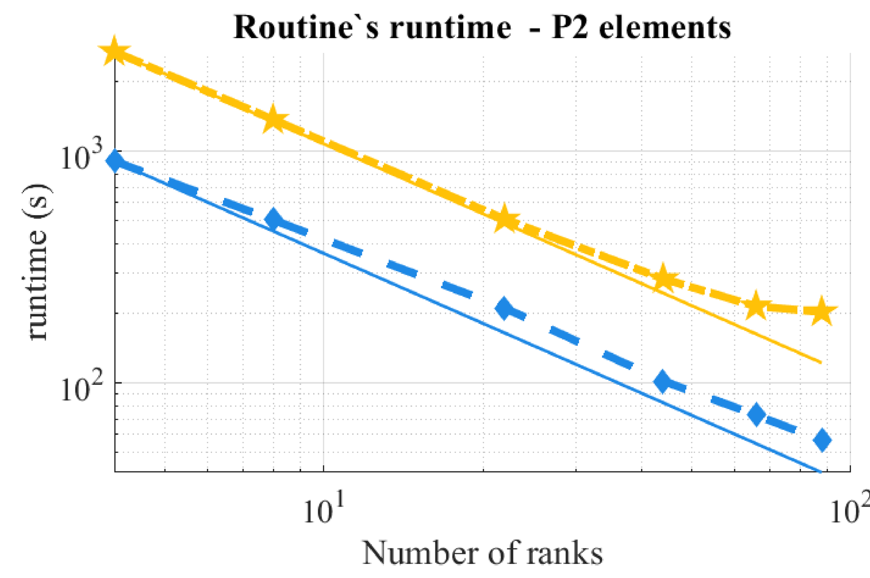


The heart mesh has 3,701,794 tetrahedral elements. The model includes 774,169 and 5,531,091 degrees of freedom per finite element field for P1 and P2 elements, respectively.



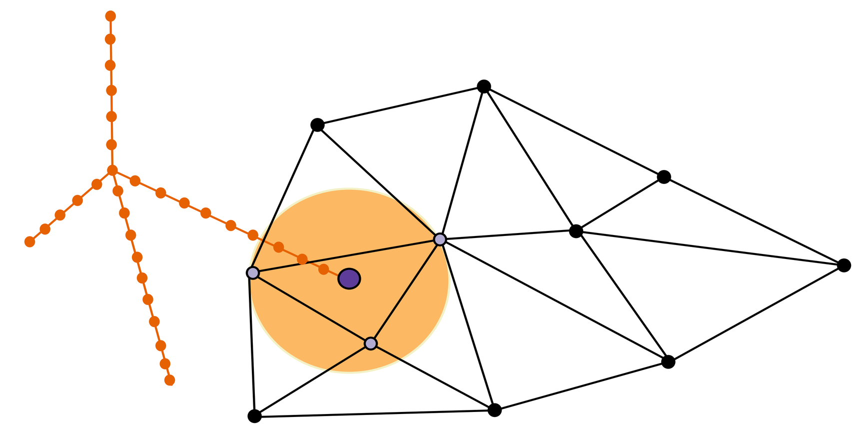




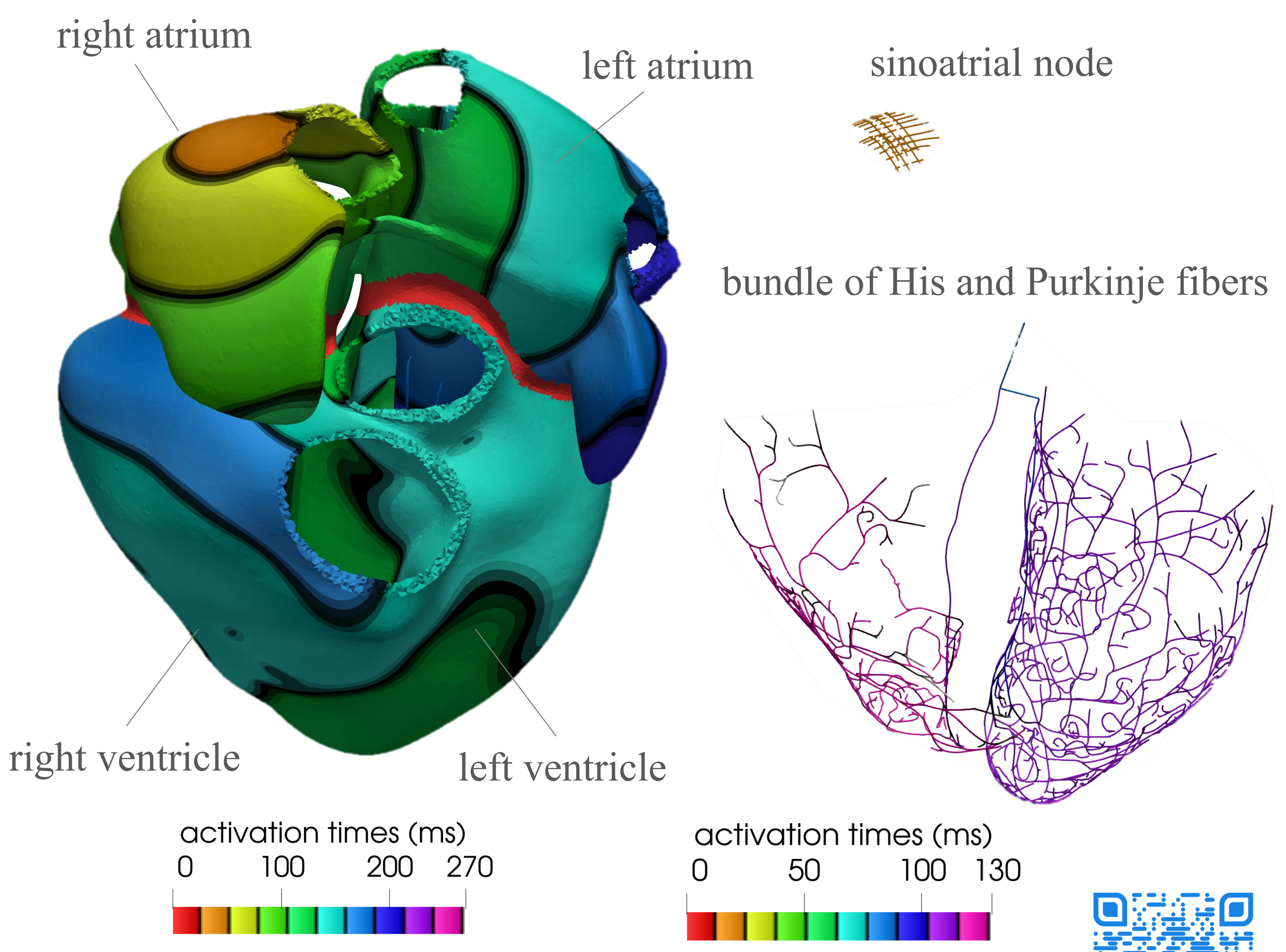


Two-way coupling among multi-dimensional meshes enables realistic activation patterns

- Coupling happens in a spherical region around the junction.
- The current from the higher-dimensional mesh is imposed as a boundary condition on the one-dimensional mesh.
- The current from the one-dimensional mesh is used as an stimulus on the intersection of the spherical region around the junction and the higher-dimensional mesh.



- coupling node (junction)
- element of one-dimensional mesh
- spherical coupling region on higher dimensional mesh
- coupling nodes in higher dimensional mesh
- element of higher dimensional mesh



activation times

Conclusions

We developed a framework that is capable of solving the monodomain equations in linear and quadratic elements. It also supports coupling between meshes with different codimensions, which is important for simulating interactions between the cardiac conduction system and the working myocardium. Finally, we performed a strong scalability study that shows promising speedup results. We aim to use this library to perform biophysically detailed electrophysiology simulations on a patient-specific heart.