Unstructured finite element models of cardiac electrophysiology using a deal.II-based library

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ABSTRACT

Organ-scale finite element models of cardiac electrophysiology require high spatial resolution to capture sharp propagating wavefronts, resulting in computationally expensive tasks. This poster presents a library specifically designed to address these demanding simulations. The library's routines support the use of linear, and quadratic tetrahedral elements. Moreover, activation of the whole heart involves models that span the thick muscular heart walls and cable-like fibers of the cardiac conduction system, which includes the bundle of his and the Purkinje network. We addressed that by enabling coupling between meshes with different codimension. By enabling such coupling, the library aims to contribute to a more comprehensive understanding of the heart's intricate electrical behavior.

CCS CONCEPTS

• **Mathematics of computing → Mathematical software performance**; • **Applied computing → Computational biology**.

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1 INTRODUCTION

Electrophysiology (EP) simulations have been used to investigate cardiovascular electrical anomalies and have the potential of guiding clinical procedure decisions [4]. As such, there has been significant effort by the scientific community in advancing the field towards patient-specific EP heart simulations [5, 7, 10]. Often, EP problems require very fine meshes to attain conduction velocity convergence. The high computational costs associated with such fine resolutions become a bottleneck for large-scale simulations [9]. To countervail that, it is common practice to adjust conductivities to yield the expected conduction velocities.

In this poster, we introduce a deal.II-based [1] library that supports linear and quadratic finite elements. We show through a benchmark simulation that using higher order finite elements may be a way to avoid using such fine resolutions. Besides, the library offers a multi-dimensional mesh coupling capability that allows for communication, through the His-Purkinje system, between atria and ventricles. Finally, we present a strong scalability study that investigates the runtime speedup as we increase the number of processors.

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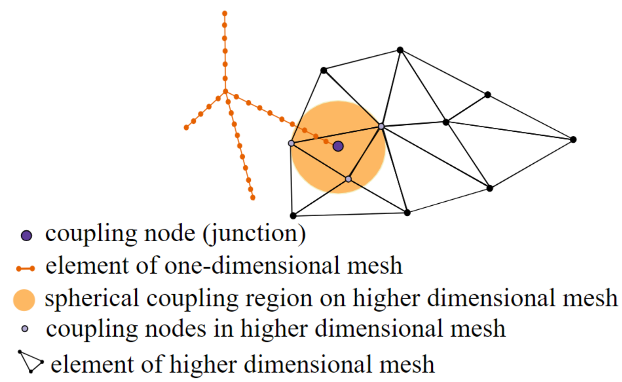

Figure 1: Coupling scheme between a one-dimensional and two-dimensional mesh.

2 METHODS

Consider the monodomain equations 1. We formulate the problem in the finite element weak form and solve for V, which stands for cell transmembrane potential difference,

$$\beta \left(C_m \frac{\partial V}{\partial t} + I_{\text{ion}}(V, w) + I_{\text{s}} \right) = \nabla \cdot (\sigma \nabla V) \quad \text{on } \Omega \tag{1}$$

$$\frac{dw}{dt} = g(V, w, t) \quad \text{on } \Omega \tag{2}$$

$$\sigma \nabla V \cdot n = 0 \quad \text{on } \partial\Omega, \tag{3}$$

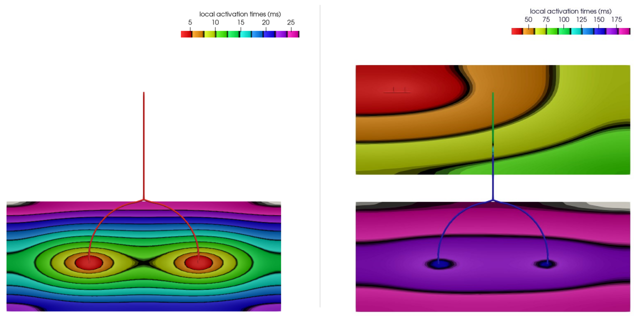

Figure 2: Local activation times. Left: One way coupling from 1D to 2D mesh. Right: Two way coupling between 1D and 2D mesh

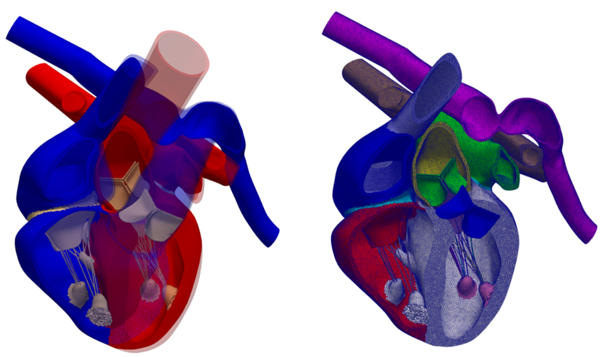

Figure 3: Patient specific heart geometry and mesh with 3,701,794 tetrahedral elements and 774,169 vertices.

in which I_{ion} is the outward transmembrane ionic current associated with phenomenological ionic models, such as Nash-Panfilov [8] and Cherry-Ehrlich-Nattel-Fenton [6]. I_s is the (initial) stimulus current, β is the ratio between the membrane area and the tissue volume, C_m is the membrane capacitance, and σ is the conductivity tensor. More specifically, we use a first order Implicit-Explicit (IMEX) temporal scheme, SBDF1 [2], Forward Euler solver for the system of ordinary differential equations associated with the ionic model, and conjugate gradient to solve the linear system associated with the partial differential equation.

When coupling meshes with different spatial dimensions, the current from the higher-dimensional mesh is imposed as a boundary condition on the one-dimensional mesh. Reversely, the current from the one-dimensional mesh is used as an stimulus on the spherical region around the junction that intersects the higher-dimensional mesh. The underlying assumption is that the coupling happens in a spherical region around the junction as shown in Figure 1. For more details on the coupling algorithm, please refer to [3].

To perform the scalability study, we used the whole heart mesh shown in Figure 3. The tests were done using two nodes and the number of cores was split equally between them whenever possible.

3 RESULTS

The results for the strong scalability study for linear and quadratic elements is shown in Figure 4. Notice that there is considerable speedup as we increase the number of processors in both cases. Since finite element problems tend to be memory bandwidth-limited, we speculate that the curves flatten out around 88 cores due to memory limitations. Therefore, we expect to get better speedup performance if we fix the number of cores per node and increase the number of nodes.

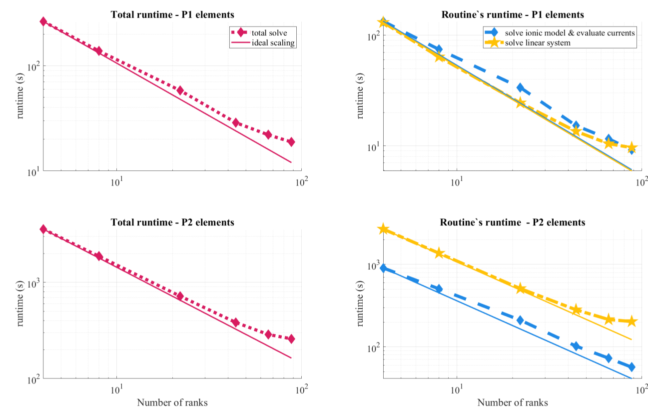

Figure 4: Runtime versus number of processors.

4 CONCLUSIONS

In this poster we introduce a deal.II-based electrophysiology library for heart simulations. The library offers implementation of phenomenological and biophysically-detailed ionic models. Besides, it offers support for use of linear and quadratic finite elements. The latter can be used to investigate the feasibility of replacing fine resolution meshes, often needed to capture conduction velocities in EP simulations, by a coarser mesh with higher-order elements. Moreover, we offer support for two-way coupling between multi-dimensional meshes for large scale electrophysiology simulations. Finally, we presented a scalability study using phenomenological ionic models that showed considerable speedups as we increase the number of computer processors. We plan to extend our scalability analysis to cases in which we use biophysically-detailed ionic models.

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